Part A: Real Numbers

Practice 1

1.

Plan

$\div (4)$

$-(8)$

Implement	Explain
$\frac{x}{4} - 8 = -3$	
$+8 \quad +8$	Addition Property of Equality
$\frac{x}{4} = 5$	
$(4)(\frac{x}{4}) = (4)(5)$	Multiplication Property of Equality
$x = 20$	

2.

Plan

$\cdot (3)$

-5

$\cdot (\frac{1}{4})$

Implement	Explain
$\frac{1}{4}(3x - 5) = -1$	
$(4)(\frac{1}{4})(3x - 5) = (4)(-1)$	Multiplication Property of Equality
$3x - 5 = -4$	
$+5 \quad +5$	Addition Property of Equality
$3x = 1$	
$(\frac{1}{3})(3x) = (\frac{1}{3})(1)$	Multiplication Property of Equality
$x = \frac{1}{3}$	

3.

$2(b - 4) + 5 = 9$

$2b - 8 + 5 = 9$

$2b - 3 = 9$

$2b - 3 + 3 = 9 + 3$

$2b = 12$

$(\frac{1}{2})2b = 12(\frac{1}{2})$

$b = 6$

4.

$$4v = 2v - 7 + 5$$
$$4v = 2v - 2$$
$$4v - 2v = 2v - 2v - 2$$
$$2v = -2$$
$$(\frac{1}{2})2v = -2(\frac{1}{2})$$
$$v = -1$$

5.

$$-2b + 5 - b + 3 = -12$$
$$-3b + 8 = -12$$
$$-3b + 8 - 8 = -12 - 8$$
$$-3b = -20$$
$$(-\frac{1}{3})\,-3b = -20(-\frac{1}{3})$$
$$b = \frac{20}{3}$$

6.

$$7(q + 2) = 9q + 5$$
$$7q + 14 = 9q + 5$$
$$7q - 7q + 14 = 9q - 7q + 5$$
$$14 = 2q + 5$$
$$14 - 5 = 2q + 5 - 5$$
$$9 = 2q$$
$$(\frac{1}{2})9 = (\frac{1}{2})2q$$
$$\frac{9}{2} = q$$

7.

$P = 26$; $l = 2x + 3$; $w = x + 4$

$P = 2l + 2w$

$$26 = 2(2x + 3) + 2(x + 4)$$
$$26 = 4x + 6 + 2x + 8$$
$$26 = 6x + 14$$
$$26 - 14 = 6x + 14 - 14$$
$$12 = 6x$$
$$12(\frac{1}{6}) = (\frac{1}{6})6x$$
$$x = 2$$

width: $x + 4 = (2) + 4 = 6$

length: $2x + 3 = 2(2) + 3 = 4 + 3 = 7$

The width of the rectangle is 6 inches and the length of the rectangle is 7 inches.

8.

$$15m + 5 = 10m + 15$$
$$15m - 10m + 5 = 10m - 10m + 15$$
$$5m + 5 = 15$$
$$5m + 5 - 5 = 15 - 5$$
$$5m = 10$$
$$(\frac{1}{5})(5m) = (\frac{1}{5})(10)$$
$$m = 2$$

Emily and Reggie each mowed two lawns.

9.

n: first integer, $n + 1$: second integer, $n + 2$: third integer

$n + (n + 1) + (n + 2) = -78$

$3n + 3 = -78$

$3n + 3 - 3 = -78 - 3$

$3n = -81$

$(\frac{1}{3})(3n) = (\frac{1}{3})(-81)$

$n = -27$

The three numbers are -25, -26, -27.

10.

Plan

$\cdot\ q$

$-\ 4$

Implement	Explain
$qx - 4 = c$	
$qx - 4 + 4 = c + 4$	Addition Property of Equality
$qx = c + 4$	
$(\frac{1}{q})(qx) = (\frac{1}{q})(c + 4)$	Multiplication Property of Equality
$x = \frac{(c + 4)}{q}$	

11.

$d = rt$; r

$d = rt$

$(\frac{1}{t})d = rt(\frac{1}{t})$

$\frac{d}{t} = 1r$

$r = \frac{d}{t}$

12.

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA - 2\pi r^2 = 2\pi r^2 - 2\pi r^2 + 2\pi rh$$
$$SA - 2\pi r^2 = 2\pi rh$$
$$\frac{SA - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$
$$h = \frac{SA - 2\pi r^2}{2\pi r}$$
$$\text{or } h = \frac{SA}{2\pi r} - r$$

Practice 2

1.

Plan

$\cdot \frac{2}{3}$

$+ 15$

Implement	Explain
$\frac{2}{3}x + 15 = -1$	
$-15 \quad -15$	Addition Property of Equality
$\frac{2}{3}x = -16$	
$(\frac{3}{2})(\frac{2}{3}x) = (\frac{3}{2})(-16)$	Multiplication Property of Equality
$x = -24$	

2.

Plan

-18

$\cdot \frac{5}{9}$

-3

Implement	Explain
$\frac{5}{9}(x - 18) - 3 = 2$	
$+3 \quad +3$	Add 3 to both sides.
$(\frac{9}{5})(\frac{5}{9})(x - 18) = (\frac{9}{5})(5)$	Multiply both sides by $\frac{9}{5}$, the reciprocal of $\frac{5}{9}$.
$x - 18 = 9$	
$+18 \quad +18$	Add 18 to both sides.
$x = 27$	

3.

$$5(2x + 3) - 8 = 65$$
$$10x + 15 - 8 = 65$$
$$10x + 7 = 65$$
$$10x + 7 - 7 = 65 - 7$$
$$10x = 58$$
$$\left(\frac{1}{10}\right)(10x) = \left(\frac{1}{10}\right)(58)$$
$$x = \frac{58}{10} = 5.8$$

In this case your student can write the solution as a decimal because dividing by 10 makes this more efficient than simplifying the fraction.

4.

$$\frac{3}{2}x - 11 = x - 4$$
$$\frac{3}{2}x - x - 11 = x - x - 4$$
$$\frac{1}{2}x - 11 + 11 = -4 + 11$$
$$\left(\frac{2}{1}\right)\left(\frac{1}{2}x\right) = \left(\frac{2}{1}\right)(7)$$
$$x = 14$$

5.

$$17 = -12x + 3 - 1 - 3x$$
$$17 = -15x + 2$$
$$17 - 2 = -15x + 2 - 2$$
$$15 = -15x$$
$$\left(-\frac{1}{15}\right)(15) = \left(-\frac{1}{15}\right)(-15x)$$
$$x = -1$$

6.

$$16(2x - 1) + 4 = -12$$
$$32x - 16 + 4 = -12$$
$$32x - 12 = -12$$
$$32x - 12 + 12 = -12 + 12$$
$$32x = 0$$
$$x = 0$$

7.

$P = 4(s)$ or $P = 2l + 2w$

$P = 96$

$s = l = w = 2x - 3$

$$96 = 4(2x - 3)$$

$$(\frac{1}{4})(96) = (\frac{1}{4})(4)(2x - 3)$$

$$24 = 2x - 3$$

$$24 + 3 = 2x - 3 + 3$$

$$27 = 2x$$

$$(\frac{1}{2})(27) = (\frac{1}{2})(2x)$$

$x = 13.5$ feet

side length $= 2(13.5) - 3$

A side length of the square is 24 feet.

8.

n: number

$$\frac{4}{5}n - 8 = \frac{3}{5}n + 14$$

$$\frac{4}{5}n - \frac{3}{5}n - 8 = \frac{3}{5}n - \frac{3}{5}n + 14$$

$$\frac{1}{5}n - 8 + 8 = 14 + 8$$

$$\frac{1}{5}n = 22$$

$$(5)(\frac{1}{5}n) = (5)(22)$$

$$n = 110$$

9.

n: first number; $n + 2$: second even number; $n + 4$: third even number

$n + (n + 2) + (n + 4) = 312$

$$3n + 6 = 312$$

$$3n + 6 - 6 = 312 - 6$$

$$3n = 306$$

$$(\frac{1}{3})(3n) = (\frac{1}{3})(306)$$

$$n = 102$$

The three numbers are 102, 104, and 106.

10.

Plan

· (10)

$-b$

Implement

$10a - b = c$

$+ b + b$

$10a = b + c$

$(\frac{1}{10})(10a) = (\frac{1}{10})(b + c)$

$a = \frac{(b + c)}{10}$

Explain

Add b to both sides of the equation.
Write variables in alphabetical order.
Divide both sides by 10.

11.

$V = lwh;\ w$

$V = lwh$

$\frac{V}{lh} = \frac{lwh}{lh}$

$w = \frac{V}{lh}$

12.

$SA = s^2 + 2sl$

$SA - s^2 = 2sl$

$\frac{SA - s^2}{2s} = \frac{2sl}{2s}$

$l = \frac{SA - s^2}{2s}$ or $l = \frac{SA}{2s} - \frac{s}{2}$

Part B: Algebraic Properties

Practice 1

1.

$2g - \frac{2}{5} = \frac{7}{5}$

LCD: 5

$5(2g - \frac{2}{5}) = 5(\frac{7}{5})$

$10g - 2 = 7$

$10g - 2 + 2 = 7 + 2$

$10g = 9$

$\frac{1}{10}(10g) = \frac{1}{10}(9)$

$g = \frac{9}{10}$

2.

$0.5h + 0.15 = 0.43h - 0.34$

Hundredth

$100(0.5h + 0.15) = 100(0.43h - 0.34)$

$50h + 15 = 43h - 34$

$7h + 15 = -34$

$7h + 15 - 15 = -34 - 15$

$7h = -49$

$(\frac{1}{7})7h = (\frac{1}{7})(-49)$

$h = -7$

3.

$\frac{5}{8}x + 1 = \frac{1}{3}(x - 2)$

LCD: 24

$24(\frac{5}{8}x + 1 = \frac{1}{3}(x - 2))$

$15x + 24 = 8(x - 2)$

$15x + 24 = 8x - 16$

$15x - 8x + 24 = 8x - 8x - 16$

$7x + 24 - 24 = -16 - 24$

$7x = -40$

$(\frac{1}{7})(7x) = (\frac{1}{7})(-40)$

$x = -\frac{40}{7}$

4.

$0.4z + 0.24 = 1.5 + 0.1z$

$100(0.4z + 0.24) = 100(1.5 + 0.1z)$

$40z + 24 = 150 + 10z$

$40z - 10z + 24 = 150 + 10z - 10z$

$30z + 24 - 24 = 150 - 24$

$(\frac{1}{30})(30z) = (\frac{1}{30})(126)$

$z = \frac{126}{30} = 4.2$

5.

$\frac{2}{3}n + 3 = \frac{1}{4}n - 1$

$12(\frac{2}{3}n + 3) = 12(\frac{1}{4}n - 1)$

$8n + 36 = 3n - 12$

$5n + 36 = -12$

$5n = -48$

$n = -\frac{48}{5}$

6.

$$4.10p + 5.26 = 15.51$$
$$100(4.10p + 5.26) = 100(15.51)$$
$$410p + 526 = 1551$$
$$-526 \quad -526$$
$$410p = 1025$$
$$p = 2.5$$

Barabara bought 2.5 pounds of fruit.

7.

Sample: An equation with no solutions will have an untrue equation in the last line. The variables will simplify out of the equation and the remaining numbers will not be equal to one another (e.g., 3 = 8).

8.

$$49 = 1 - (x + 1)$$
$$49 = 1 - x - 1$$
$$49 = -x$$
$$x = -49$$

This equation has one solution.

9.

$$5(2x + 3) = 8x - (-2x + 6)$$
$$10x + 15 = 8x + 2x - 6$$
$$10x + 15 = 10x - 6$$
$$10x - 10x + 15 = 10x - 10x - 6$$
$$15 = 6$$

This equation has no solution, $\varnothing$.

10.

$$\frac{1}{4}(12x - 8) = 3x - 2$$
$$3x - 2 = 3x - 2$$
$$3x - 3x - 2 = 3x - 3x - 2$$
$$-2 = -2$$

The equation has a solution of all real numbers, R.

11.

(clear the decimals)

$$0.70f + 0.71 = 0.70f + 0.68$$
$$100(0.70f + 0.71) = 100(0.70f + 0.68)$$
$$100(0.70f) + 100(0.71) = 100(0.70f) + 100(0.68)$$
$$70f + 71 = 70f + 68$$
$$70f - 70f + 71 = 70f - 70f + 68$$
$$0 + 71 = 0 + 68$$
$$71 = 68$$

Solution: No solution

12.
Your student should construct an equation that results in the variables being eliminated and the remaining numbers being equal to one another.

Practice 2

1.
$\frac{2}{3}x - 3 = \frac{7}{3}x + 8$
LCD: 3
$3(\frac{2}{3}x - 3 = \frac{7}{3}x + 8)$
$2x - 9 = 7x + 24$
$2x - 2x - 9 = 7x - 2x + 24$
$-9 - 24 = 5x + 24 - 24$
$-33 = 5x$
$(\frac{1}{5})(-33) = (\frac{1}{5})(5x)$
$x = -\frac{33}{5}$

2.
$0.1h - 0.2 = 1.4$
"Tenth"
$10(0.1h - 0.2) = 10(1.4)$
$1h - 2 = 14$
$h - 2 = 14$
$h - 2 + 2 = 14 + 2$
$h = 16$

3.
$-\frac{2}{3}x + \frac{5}{4} = \frac{1}{2}$
LCD: 12
$12(-\frac{2}{3}x + \frac{5}{4}) = 12(\frac{1}{2})$
$-8x + 15 = 6$
$-8x = -9$
$1x = \frac{9}{8}$
$x = \frac{9}{8}$

4.

$$5.75 - 1.50 - 0.17p = 0$$
$$4.25 - 0.17p = 0$$
$$100(4.25 - 0.17p) = 100(0)$$
$$425 - 17p = 0$$
$$425 - 17p + 17p = 0 + 17p$$
$$425 = 17p$$
$$\left(\frac{1}{17}\right)(425) = \left(\frac{1}{17}\right)(17p)$$
$$p = 25$$

5.

n: number

$$\frac{5}{4}n - \frac{21}{2} = \frac{1}{5}n$$

LCD: 20

$$20\left(\frac{5}{4}n - \frac{21}{2} = \frac{1}{5}n\right)$$
$$25n - 210 = 4n$$
$$25n - 25n - 210 = 4n - 25n$$
$$-210 = -21n$$
$$\left(-\frac{1}{21}\right)(-210) = \left(-\frac{1}{21}\right)(-21n)$$
$$n = 10$$

6.

Let c = the number of students in each class.

$$\frac{3}{8}c + 2 = \frac{1}{4}c + 5$$

LCD: 8

$$3c + 16 = 2c + 40$$
$$c + 16 = 40$$
$$c = 24$$

There are 24 students in each class.

7.

Sample: An identity will result in an equation that is always true. The variables will simplify out of the equation and the remaining numbers will be equal to one another (e.g., 5 = 5).

8.

$$2(b - 4) = 2b - 8$$
$$2b - 8 = 2b - 8$$
$$0 - 8 = 0 - 8$$
$$-8 = -8$$

The equation has a solution of all real numbers for b, R.

9.

$3p + 5 = 5p - 2p - 2$
$3p + 5 = 3p - 2$
$0 + 5 = 0 - 2$
$5 = -2$

The equation has no solution, $\varnothing$.

10.

$2n + 7 = 3n - 2$
$2n - 2n + 7 = 3n - 2n - 2$
$7 = n - 2$
$7 + 2 = n - 2 + 2$
$n = 9$

This equation has one solution.

Check:

$2(9) + 7 = 3(9) - 2$
$18 + 7 = 27 - 2$
$25 = 25$

11.

$\frac{6}{7} - \frac{2}{3}v + \frac{1}{7} = \frac{2}{3}v + \frac{3}{7} - 1\frac{1}{3}v + \frac{4}{7}$
$-\frac{2}{3}v + 1 = -\frac{2}{3}v + 1$
$3(-\frac{2}{3}v + 1) = 3(-\frac{2}{3}v + 1)$
$3(-\frac{2}{3}v) + 3(1) = 3(-\frac{2}{3}v) + 3(1)$
$-2v + 3 = -2v + 3$
$-2v + 2v + 3 = -2v + 2v + 3$
$0 + 3 = 0 + 3$
$3 = 3$

This equation has a solution of all real numbers, R.

12.

Your student should construct an equation that results in the variables being eliminated and the remaining numbers being unequal to one another.

Targeted Review

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Lesson Origin	1	1	1	1	1	1	1	1	1	PA	PA	PA	PA	PA	1	1

1.
$\{\mathbb{Q}, \mathbb{R}\}$

2.
$\{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

3.
rational $(5 + \frac{1}{3})$

4.
irrational

5.
Identity Property of Addition

6.
Addition Property of Equality

7.
Distributive Property, Zero-Product Property

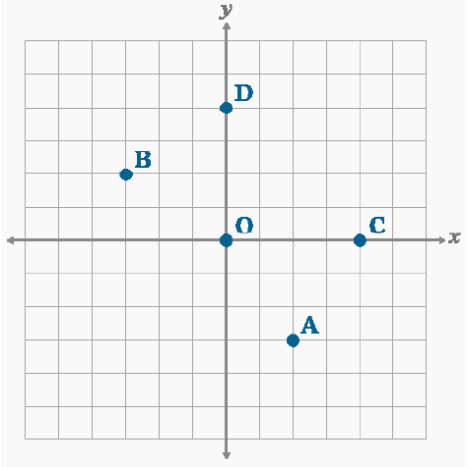
8.
Commutative Property, Associative Property

9.


10.
$m + |p - m| - p$
$(6) + |(-4) - (6)| - (-4)$
$6 + |-10| + 4$
$6 + 10 + 4$
20

11.
$-|mp| + m$
$-|(6)(-4)| + (6)$
$-|-24| + 6$
$-24 + 6$
-18

12.



13.
P: perimeter, l: length, w: width
$P = 18$, $l = 6$
$P = 2l + 2w$
$18 = 2(6) + 2w$
$18 = 12 + 2w$
$18 - 12 = 12 - 12 + 2w$
$6 = 2w$
$(\frac{1}{2})(6) = (\frac{1}{2})(2w)$
$3 = w$
The width of the rectangular garden is 3 feet.

14.
V: volume, h: height, w: width, l: length
$V = 24$, $h = 2$, $w = 2$
$V = lwh$ – Formula for the volume of a rectangular prism
$24 = (l)(2)(2)$
$24 = 4(l)$
$(\frac{1}{2})(24) = (\frac{1}{2})(4l)$
$l = 6$
The length of the box is 6 feet.

15.

☐ $\sqrt{4} + \sqrt{5}$ Q + I = I

☑ $\frac{1}{2} + \frac{11}{13}$ Q + Q = Q

☐ $\pi \cdot \pi$ I · I = I

☑ $\frac{8}{\pi} \cdot \pi$ Q · Q = Q (π cancels out leaving the rational value 8.)

16. C

120% of 90

1.2 (90) = 108

A) 18 pages This is 20% of 90 but not 120% of Samson.

B) 20 pages This is the percent without the % symbol.

C) 108 pages

D) 110 pages This is 90 + 20.